

# **MATH REVIEW SHEETS**

## **BEGINNING ALGEBRA MATH 60**

### **A Summary of Concepts Needed to be Successful in Mathematics**

The following sheets list the key concepts which are taught in the specified math course. The sheets present concepts in the order they are taught and give examples of their use.

### **WHY THESE SHEETS ARE USEFUL -**

- To help refresh your memory on old math skills you may have forgotten.
- To prepare for math placement test.
- To help you decide which math course is best for you.

### **HOW TO USE THESE SHEETS -**

- Students who successfully review spend from four to five hours on this material. We recommend that you cover up the solutions to the examples and try working the problems one by one. Then, check your work by looking at the solution steps and the answer.

### **KEEP IN MIND -**

- These sheets are not intended to be a short course. You should use them to simply help you determine at what skill level in math you should begin study. For many people, the key to success and enjoyment of learning math is in getting started at the right place. You will, most likely, be more satisfied and comfortable if you start onto the path of math and science by selecting the appropriate beginning stepping stone.

### **I. Maintain, use, and expand skills and concepts learned in previous mathematics courses.**

#### **A. Perform operations with fractions and decimals.**

- 1) Perform the following operations and simplify, if possible:

a)  $\frac{2}{3} + \frac{5}{8}$

b)  $\frac{2}{3} \div \frac{5}{8}$

c)  $3.6 + 8.95 - 4.07$

**B. Use unit analysis to convert units and solve problems. (Note: You may use the Units of Measure handout.)**

- 2) How many milliliters are there in a gallon of milk?
- 3) A swimmer can swim 10 laps in 18 minutes. If one lap is 50 meters long, how fast is the person swimming, in miles per hour.

**II. Perform addition, subtraction, multiplication, and division of rational numbers.**

- 4) Perform the following operations and simplify, if possible:

a)  $-6 + (-4) - (-8)$

b)  $5 - 4 \cdot 3$

c)  $2[3 - 3(-5 + 3)] - (-1)$

d)  $\left(-\frac{12}{25}\right) \div \left(-\frac{4}{15}\right)$

e)  $-\frac{3}{4} + \left(-\frac{5}{8}\right)$

f)  $\frac{-3(2 + (-5))}{1 - (-6)(1 - 4)}$

- 5) a) You have \$368.53 in your account. You write checks for \$183.45 and for \$201.01. What is your new account balance?
- b) You now make a deposit of \$325.89. What is your account balance now?

**III. Use and apply the concepts and language of algebraic expressions.****A. Use variables to construct algebraic expressions.**

- 6) Translate each phrase into a mathematical expression.
  - a) a number increased by 12
  - b) 4 times the quotient of a number and 5
  - c) the total cost of  $x$  bags of Halloween candy if each bag costs \$3.75
  - d) the total amount paid for an item that costs  $x$  dollars, including a 5% sales tax
- 7) If the smaller of two consecutive even integers is  $x$ , express the larger number in terms of  $x$ .
- 8) If an 8 ft. piece of lumber is cut into two pieces, and the length of one of the pieces is  $x$ , write an expression for the length of the other piece in terms of  $x$ .

**B. Evaluate algebraic expressions and simplify expressions using order of operations.**

- 9) Evaluate and simplify each expression:
  - a)  $-(-x)$  when  $x = -2$
  - b)  $xy(2 - 3x)$  when  $x = -1$  and  $y = 3$
  - c)  $\frac{-3 - z}{4 - w}$  when  $z = -5$  and  $w = 8$
  - d)  $\frac{5}{9}(F - 32)$  when  $F = -22$

**C. Simplify algebraic expressions by removing parentheses and combining like terms.**

- 10) Remove parentheses if necessary and simplify:

- a)  $4(2x - 3)$                       b)  $-6(3 - x)$                       c)  $-5(-6m)$   
 d)  $3.7x - 7.5y + 9 - 4.3y + 2$       e)  $2 - (x + 3)$                       f)  $6 - \frac{2}{3}(x + 3) + \frac{5}{6}x$

#### IV. Solve linear equations and inequalities.

##### A. Solve linear equations and formulas algebraically.

11) Solve each equation algebraically. Remember that you can check your solutions by substitution.

- a)  $2x + 4 = 8$                       b)  $5 - 4x = 17$                       c)  $\frac{-x}{5} = 3$   
 d)  $-5y = 3y$                       e)  $-3(2x - 8) = -12$                       f)  $4 + 3(x + 2) = 12 - 5x$   
 g)  $4.3(2x - 3) + 2.8 = 9.5x - 3$                       h)  $\frac{2}{3}x - 5 = -9$   
 i)  $P = 2l + 2w$  for  $w$                       j)  $V = \frac{1}{3}Bh$  for  $h$

##### B. Solve linear inequalities and graph their solutions on a number line.

12) Write down two values of  $x$  that are solutions of the inequality  $3 - 2x \leq 1$  and two values of  $x$  that are not solutions. Is  $x = 1$  a solution? Why or why not?

13) Solve each inequality and graph your solution on a number line.

- a)  $3x < 12$                       b)  $-3x \leq 12$   
 c)  $5x - 1 \geq 14$                       d)  $3 - 2x > 10$

#### V. Use algebra to solve application problems.

##### A. Translate verbal models into algebraic expressions and/or equations. Solve problems.

14) One number is 3 more than twice another number. If the sum of the numbers is 57, find the numbers.

15) You are taking a date to dinner and plan to pay for both meals, but you only have \$32. If you plan to leave a 20% tip, how much can you spend on the meals at most?

##### B. Solve problems involving simple interest, motion, and mixtures.

16) How fast must a plane fly to travel 2500 miles in 4.5 hours?

17) A man is walking at a rate of 3 miles per hour. How far will he walk in 36 minutes?

18) Suppose you have invested a total of \$12,000 in two savings accounts. One account earns 5% simple interest and the other earns 8% simple interest. Find the amount invested in each account if you receive a total of \$840 interest after 1 year.

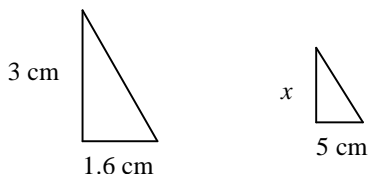
- 19) A chemist has a 25% hydrochloric acid solution and a 50% hydrochloric acid solution. How many liters of each should she mix to get 9 liters of solution with a 40% acid concentration?

**C. Solve problems using ratios and proportions.**

- 20) The number of cell phones in use worldwide in 1987 was 2.5 million. By 1997 the number of cell phones in use had increased to 213.8 million worldwide. a) Write the ratio of number of cell phones in use in 1997 to the number in use in 1987. b) Write this ratio as some quantity to 1, and explain what this means.
- 21) Solve each proportion.
- a)  $\frac{x}{9} = \frac{8}{20}$                       b)  $\frac{6}{5} = \frac{-12}{x}$                       c)  $\frac{y+3}{y-2} = \frac{5}{8}$
- 22) Set up and solve a proportion to answer this question. If a 4-ounce piece of sponge cake has 160 calories, how many calories would you expect a 9-ounce piece to have?
- 23) Set up and solve a proportion to answer this question. On Mars, 3.5 ergles are equivalent to 2 peks. How many ergles are there in 27 peks?

**D. Solve problems involving similar triangles.**

- 24) The triangles in the pair of diagrams below are similar. Find the value of  $x$ .



- 25) Wilson is 6 feet tall. When Wilson stands 6 feet from a lamppost, the shadow he casts is 4 feet long. How tall is the lamppost?

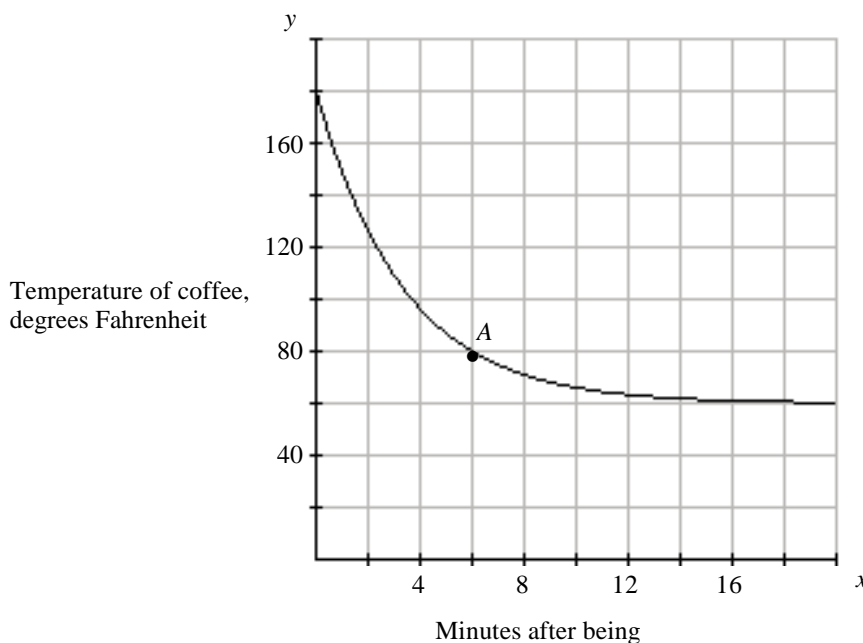
**E. Solve geometry problems involving perimeter, area, and volume.**

- 26) A triangle has a base of 6 feet and an area of 40 square feet. What is the height of the triangle?
- 27) The circumference of a circle is  $16\pi$  inches. Find the circle's radius and diameter.
- 28) What happens to the area of a circle if its radius is doubled?
- 29) A rectangular swimming pool measures 14 feet by 30 feet. The pool is surrounded on all four sides by a path that is 3 feet wide. If the cost to resurface the path is \$2 per square foot, what is the total cost of resurfacing the path?

**VI. Interpret information represented numerically and graphically, and recognize linear relationships represented verbally, numerically, graphically, and algebraically.**

**A. Read and interpret information given in a table or graph.**

- 30) The graph shows the temperature,  $y$ , of a cup of coffee  $x$  minutes after it was poured. Use the graph to answer the following questions.



- What are the coordinates of the point labeled A?
- Write a sentence describing the meaning of the point labeled A.
- What was the temperature of the coffee when it was first poured?
- After a while, the temperature of the coffee levels off and becomes the same as the outside temperature. What is the outside temperature?

**B. Locate points in a rectangular coordinate system and represent equations in two variables graphically.**

- 31) Plot each of the following points in a Cartesian coordinate system and tell which quadrant each point falls in, if any:

a)  $A(5, 3)$    b)  $B(0, 6)$    c)  $C(5, -0.5)$    d)  $D(-4, 3)$    e)  $E(-6, -1)$    f)  $F(-2, 0)$

- 32) Sketch a careful graph of each of the following equations on a sheet of graph paper.

a)  $y = 2x - 5$    b)  $y = \frac{1}{3}x + 2$    c)  $3x - 4y = 6$

- Graph the line that passes through the point  $(0, 3)$  and has a slope of  $-2$ .
- Graph the line that passes through the point  $(-1, -4)$  and has a slope of  $3/2$ .
- Graph the line that passes through the point  $(2, 3)$  and has an undefined slope.

**C. Identify the horizontal and vertical intercepts of the graph of an equation and interpret them in terms of an application.**

36) Find the  $x$ -intercept and the  $y$ -intercept of each of the following linear relationships. Write your answers as ordered pairs.

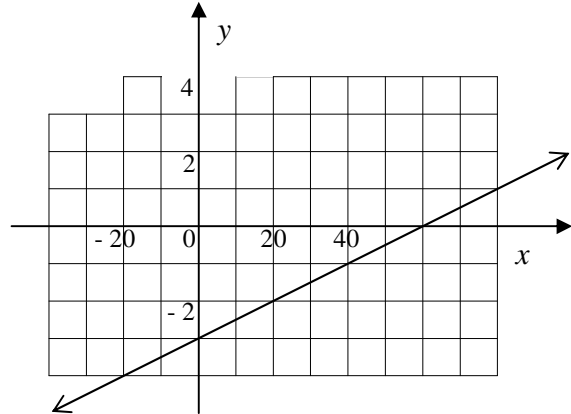
a)  $3x + 2y = 12$

b)  $y = -2x + 5$

c)

$x$	$y$
-1	4
0	2
1	0
2	-2

d)



**D. Identify the slope of a line and interpret it in terms of an application.** (You should

remember the slope formulas:  $m = \frac{y_2 - y_1}{x_2 - x_1}$ ,  $m = \frac{\Delta y}{\Delta x}$  and  $m = \frac{\text{rise}}{\text{run}}$ .)

37) Find the slope of the line that passes through each pair of points.

a)  $(6, 3)$  and  $(-5, 2)$

b)  $(-2, -1)$  and  $(0, 8)$

c)  $(5, 9)$  and  $(5, -3)$

38) Find the slope of the line represented in the table in problem #36c above.

39) Find the slope of the line in problem #36d above.

40) What is the slope of every vertical line? Every horizontal line?

**E. Use the slope-intercept form of the equation of a line.** (Remember:  $y = mx + b$ .)

41) Write each of the following equations in slope-intercept form and state the slope and  $y$ -intercept.

a)  $3x + 2y = 12$

b)  $1 - 3x = y$

c)  $2x = -5y$

42) Write the equation of a line with slope  $-5$  and  $y$ -intercept  $(0, -3)$ .

43) Write the equation of a line with zero slope and  $y$ -intercept  $(0, 4)$ .

44) Write the equation of the line represented in the table in problem #36c above.

45) Write the equation of the line represented in the graph in problem #36d above.

46) A rental car company charges \$120 for a weekly rental plus 8 cents per mile driven. Use this information to answer the following questions:

a) Complete a table of values showing the relationship between  $x$ , the number of miles driven, and  $y$ , the total amount charged for a one-week rental. Use  $x = \{0, 100, 200, \dots, 600\}$ .

- b) Use your table above to give a reasonable estimate for the total amount charged for driving 700 miles.
- c) On a sheet of graph paper, graph the relationship using your table of values.
- d) Find the slope of this line, and interpret its meaning in this application.
- e) Find the  $y$ -intercept of this line, and interpret its meaning in this application.
- f) Write the equation of this line.
- g) Use the equation to determine the number of miles that can be driven during the week for \$250.

**VII. Make appropriate and efficient use of a scientific calculator. (Note: Students will be expected to demonstrate achievement of some objectives *without* the use of a calculator.)**

## Answers for the Math 60 Review Objectives and Sample Problems

- 1) a)  $\frac{31}{24}$     b)  $\frac{16}{15}$     c) 8.48
- 2) 3784.3 mL
- 3) 1.04 miles per hour
- 4) a) -2    b) -7    c) 19  
d)  $\frac{9}{5}$     e)  $-\frac{11}{8}$     f)  $-\frac{9}{17}$
- 5) a) -\$15.93    b) \$309.96
- 6) a)  $x+12$     b)  $4 \cdot \left(\frac{x}{5}\right)$   
c)  $3.75x$     d)  $x+0.05x=1.05x$
- 7)  $x+2$
- 8)  $8-x$
- 9) a) -2    b) -15    c)  $-\frac{1}{2}$     d) -30
- 10) a)  $8x-12$     b)  $-18+6x$   
c)  $30m$     d)  $3.7x-11.8y+11$   
e)  $-1-x$     f)  $4+\frac{1}{6}x$
- 11) a)  $x=2$     b)  $x=-3$   
c)  $x=-15$     d)  $y=0$   
e)  $x=6$     f)  $x=\frac{1}{4}$   
g)  $x \approx -7.89$     h)  $x=-6$   
i)  $w = \frac{P-2l}{2}$   
j)  $h = \frac{3V}{B}$
- 12) *examples:*  $x=4$  and  $x=5$  are solutions.  
 $x=-3$  and  $x=-2$  are not solutions.  
 $x=1$  is a solution because  $1 \leq 1$ .
- 13) a)  $x < 4$     b)  $x \geq -4$   
c)  $x \geq 3$     d)  $x < -\frac{7}{2}$
- 14) 18 and 39
- 15) \$26.67
- 16) 555.6 miles per hour
- 17) 1.8 miles
- 18) \$4000 @ 5%  
\$8000 @ 8%
- 19) Mix 3.6 liters of 25% HCl solution with 5.4 liters of 50% HCl solution.
- 20) a)  $\frac{213.8}{2.5}$     b)  $\frac{85.52}{1}$   
There were about 85.52 times as many cell phones in use in 1997 as in 1987.
- 21) a)  $x = \frac{18}{5}$     b)  $x = -10$   
c)  $y = -\frac{34}{3}$
- 22) 360 calories
- 23) 47.25 ergles
- 24)  $x = 9.375$  cm
- 25) 15 feet
- 26)  $\frac{40}{3}$  feet
- 27) radius = 8 inches, diameter = 16 inches



28) The area is quadrupled.

29) \$600

30) a) (6, 80)

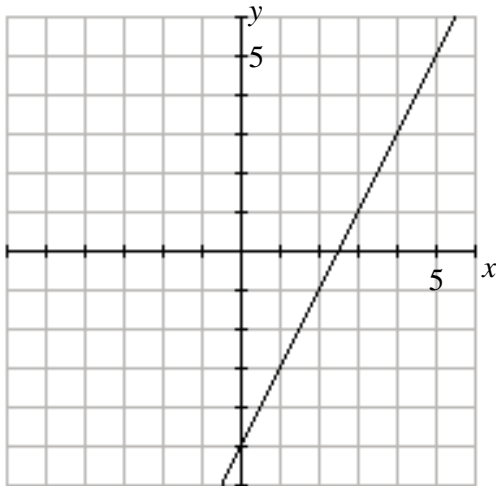
b) Six minutes after being poured, the temperature of the coffee is 80 degrees.

c) about 180 degrees Fahrenheit

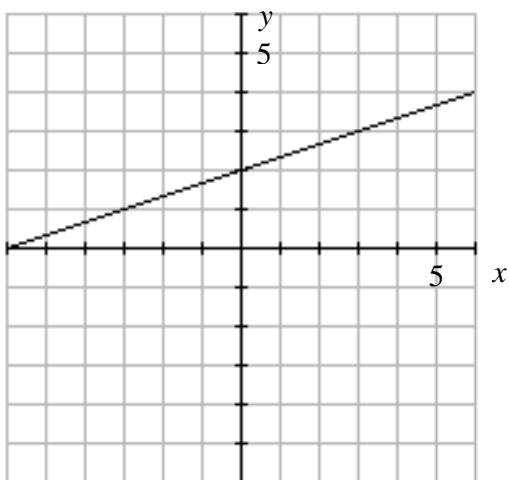
d) about 60 degrees Fahrenheit

31) a) I    b) y-axis    c) IV  
 d) II    e) III    f) x-axis

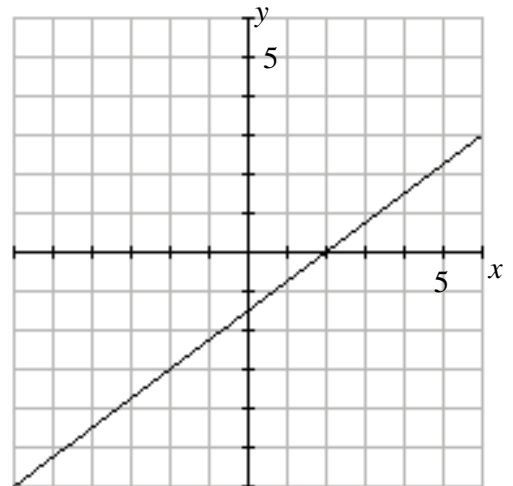
32) a)



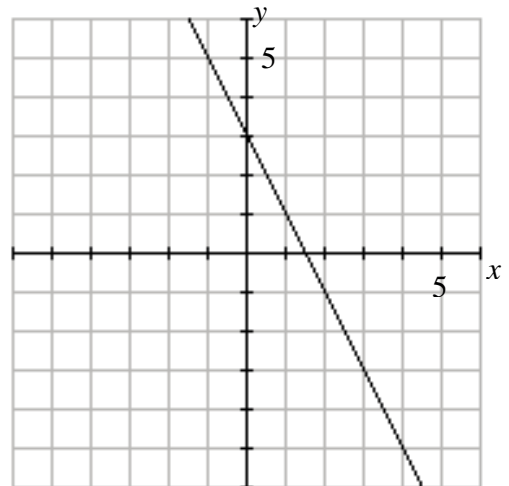
32) b)



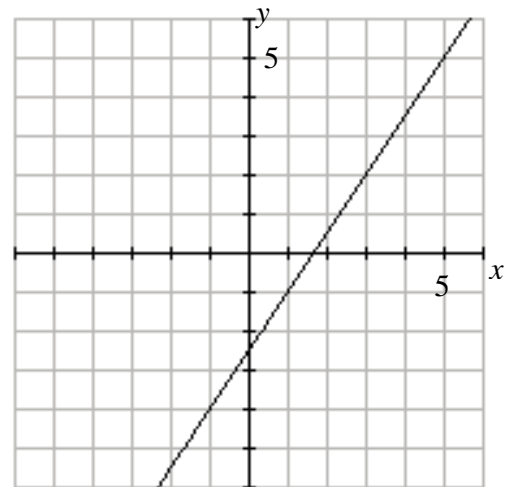
32) c)



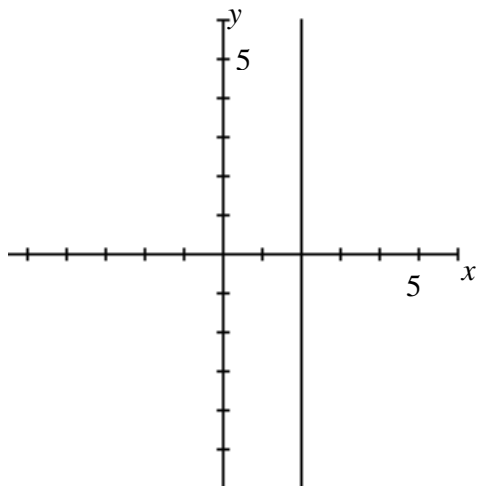
33)



34)



35)



43)  $y = 4$

44)  $y = -2x + 2$

45)  $y = \frac{1}{20}x - 3$

46) a)

$x$	$y$
0	120
100	128
200	136
300	144
400	152
500	160
600	168

36) a)  $x$ -int: (4, 0);  $y$ -int: (0, 6)

b)  $x$ -int: (5/2, 0);  $y$ -int: (0, 5)

c)  $x$ -int: (1, 0);  $y$ -int: (0, 2)

d)  $x$ -int: (60, 0);  $y$ -int: (0, -3)

37) a) slope = 1/11

b) slope = 9/2

c) slope is undefined

38) slope = -2

39) slope =  $3/60 = 1/20$

40) Vertical lines have undefined slope, and horizontal lines have zero slope.

41) a)  $y = -\frac{3}{2}x + 6$ ; slope =  $-\frac{3}{2}$ ;  $y$ -int: (0, 6)

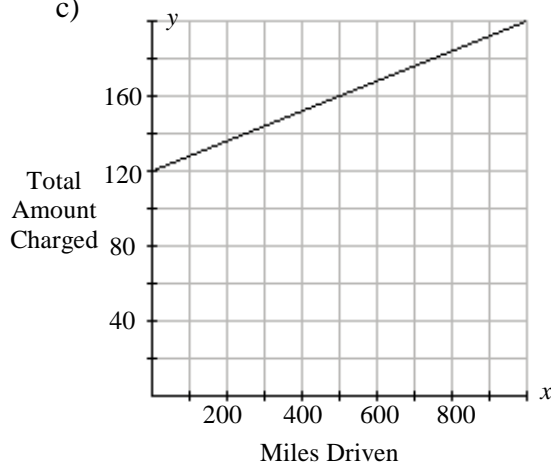
b)  $y = -3x + 1$ ; slope = -3;  $y$ -int: (0, 1)

c)  $y = -\frac{2}{5}x$ ; slope =  $-\frac{2}{5}$ ;  $y$ -int: (0, 0)

42)  $y = -5x - 3$

b) \$176

c)



d) slope = 0.08

This is the cost per mile driven:  
\$0.08 per mile.

e)  $y$ -int: (0, 120)

If you drive no miles, you'll be charged  
\$120. This is the weekly charge.

f)  $y = 0.08x + 120$

g) 1625 miles