Math and Music Connections

Sabbatical Report by Amy Gaudia, © June 2019

Math and Music Connections: An exploration of how basic mathematical skills may be improved by incorporating music and musical concepts into the Adult Basic Skills math classroom

Part 1: Fretting About Frets

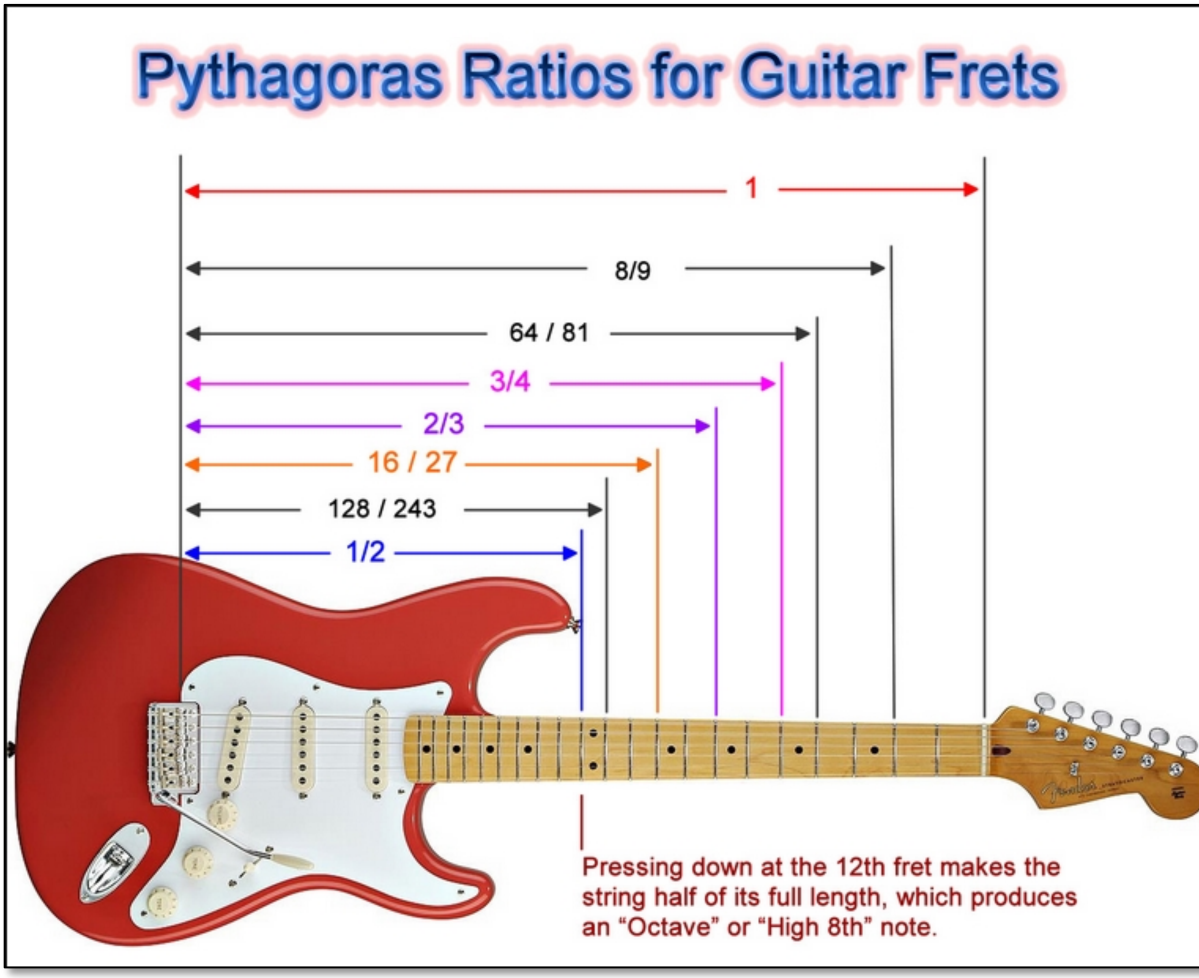
"All things which can be known have number; for it is not possible that without number anything can either be conceived or known." Philolaus in the fourth century BC

I have been teaching General Education Development (GED) classes and coordinating tutors in the Adult Basic and Secondary Education (ABSE) department for over 23 years, and have experienced both the excitement and the challenge of teaching and interacting with students at all skill levels in math and language arts. My assignment for the last four years has been Math 1 and I am constantly surprised to find that many of these students are lacking the most basic arithmetic skills. Like most instructors, I am always working to improve my instruction and to find new and better ways to help my students access their learning potential and gain the skills they need to move forward toward earning their GEDs and/or placing into credit classes.

One positive and inspiring aspect of working with this population is that many of them are very open to trying new things and giving themselves a chance to understand the interesting mathematical connections that I have pointed out to them. For example, when showing them a spiral made from the Fibonacci series, they became more attentive and curious about the lesson that followed. I have also briefly and spontaneously talked about musical rhythms as they relate to some basic fractions, and found the same to be true – greater involvement in the class lesson and other activities.

And thus I started exploring the possibility of incorporating musical concepts into this basic math class.

I found this wonderful image to post on my office door:



Every day I would arrive at my office in the early morning, and study the image while unlocking the door. I knew what I was looking at. After all, I have been playing the guitar for 45 years including an informal study of music theory. I definitely had a thorough understanding that the halfway point along a guitar fret board produced a sound an octave higher than the open string as well as the corresponding pitch frequencies. But all the other numbers on this image were puzzling. I had a general idea of what it was all about but I could not make sense of all those ratios. Instead of researching it, I was certain that if I just stared at the image and pondered it long enough, I could figure it all out. Could this be what some of my students are doing in class?

And so the first weeks of my sabbatical project were devoted to placing myself in the role of student with a swirling cloud of numbers before me, and a determination to solve the mystery. I read several accounts of the tuning system developed by the Greek mathematician Pythagoras and although this is sometimes credited to some of his contemporaries, it is usually referred to as Pythagorean tuning. He believed that music had a divine relationship with numbers. The story is told that when he heard the different sounds produced by the hammers of the blacksmiths working nearby, he was curious about the frequencies of the pitches and how some of the pitches sounded very nice together. He learned that the masses of the hammers were in simple integer ratios, which he then attempted to duplicate with strings attached to a block of wood. He theorized that all consonant sounds, notes that sound pleasant or agreeable to the human ear, have frequencies in simple small integer ratios with one another. He first divided the string in half and found that it made a beautiful and consonant sounding pitch. It is known today as a note that is an octave higher, and can be compared to voices when a young child and a man can sing together on the same note. When he then divided a string at 2/3 the length of the original, it sounded beautiful when played simultaneously with the original full-length string. In fact, it was so lovely that today it is called a "perfect fifth." In Western music, this is the fifth note of an 8 note, or diatonic, scale. Pythagoras continued building his scale by stacking the 2/3 and 1/2 ratios in a

process that does eventually reveal the other ratios that were confusing to me. For example, he wanted a note that would fit within the scale of the low and high octave and have the same 2/3 ratio with the high octave. So he took the ½ length string and multiplied by the inverse of 2/3. (He used the inverse because he was creating a string that would be longer that the ½ length string.) $1/2 \times 3/2 = 3/4$
And so a string with a length ¾ of the original string became what is known today as the fourth note in the diatonic scale.

The guitar image was still puzzling me for another reason. With some basic knowledge of pitch frequencies, and harmonics, I wondered how those Pythagorean ratios are used to measure the distance of all the frets on modern guitars. My understanding was that the ratios of Pythagoras resulted in a system of notes and scale building that would eventually not work out for transposing to other keys and other octaves. For example, the interval created by the first and third notes of his scale (now known as a major third) were slightly off and dissonant. And so, equal temperament tuning was developed to remedy those problems by shortening and lengthening the Pythagorean intervals as needed. According to the *Encyclopedia Britannica* and *Oxford Music Online*, the theory of equal temperament was first published in 1584 by Chu Tsai-Yu of the Ming dynasty. It grew in popularity during the following two centuries, and became quite favored by some composers such as J.S. Bach, well known for his work *The Well-Tempered Clavier,* a collection of preludes and fugues written in all 24 keys. There are many tuning systems found in other parts of the world that build scales of smaller intervals, or spaces between the notes. In Indian theory, the octave is divided into 22 tones, or srutis. This will be a great subject for further investigation.

In *Mathematics, Music, and the Guitar* by David Hornbeck, the author explains in great detail, how the method used by Pythagoras resulted in adjacent notes of the scale not being equally spaced. This made it difficult to transpose music. Transposing is sometimes needed for music to be played or sung in a higher or lower key. He goes on to illustrate the development of the formula used in 12 tone equal temperament (12 TET) tuning.

$$\frac{log_2(2^{\frac{m}{12}}) - log_2(2^{\frac{n}{12}})}{m-n}$$
$$= \frac{\frac{1}{12}(m-n)}{m-n} = \frac{1}{12}$$

Although this equation is far above the level taught in my Math 1 class, the idea of guitar building as part of a math curriculum is quite appropriate. The National Science Foundation STEM Guitar Project has been doing this for over 8 years.

"During the NSF grant cycles, the STEM Guitar Project has exceeded initial estimates of faculty impacted by recruiting over 450 STEM educators, with an additional 500 faculty exposed via national education conferences. Thus far, this effort is impacting over 20,000 students nationally over the 8 years because of faculty members adopting or adapting the curriculum developed through the project."

My first introduction to the STEM Guitar project is what inspired me to try my own didgeridoo building project, which in turn resulted in my interest in further exploring the possibilities of involving music in the basic skills math class. The didgeridoo is an indigenous Australian hollow tube wind instrument. In the project, the music concept that we experienced was pitch and its relationship to the length of the tubes. In the future I would like to expand the project into building a simple marimba, a percussion instrument with a set of wooden bars like a keyboard. I predict this will be a very engaging way for students to study Pythagorean tuning and the concept of ratios.

In my personal musical life, I now have a greater understanding of the difficulty we musicians often face in tuning our instruments and why music can sometimes sound better in one key compared to another key. I also have a deeper understanding of how these challenges are experienced differently on my fretted instrument, the guitar, compared to the (unfretted) cello.

This is the formula presently used for fret spacing on guitars:

$D_n = [(L - D_{n-1}) \div 17.817] + D_{n-1}$

https://archive.siam.org/careers/pdf/guitar.pdf

Part II: The Quadrivium

"Music is so naturally united with us that we cannot be free from it even if we so desired."
--Boethius

My interest in the Quadrivium, the four classical liberal arts of number, geometry, music and cosmology is largely due to its focus on the soul or in my terms, the "whole being." During the Middle Ages and right on through the Renaissance, people attributed everything to the divine and found meaningful connections between numbers, beauty, planetary motions, sounds, and all of the natural world. Dr. Christopher Perrin of Classical Academic Press describes classical education as "the cultivation of affections or loves" and "the cultivation of the soul on truth, goodness and beauty."

In my work teaching basic skills to adults, I have found many students to be very tuned in to philosophical ideas, spirituality, mysticism, and personal growth topics. My experience with them has led me to believe that something “opens up” and they become more present and involved when making mathematical connections to art, music, beauty, and much more. Health and nutrition, social activism, and many other aspects of life become more interesting. Imagine that you are 28 years old, sitting in a windowless classroom with old worn out walls, trying to memorize basic multiplication facts! Something is greatly needed.

It is Boethius (Saint Anicius Manlius Severinus Boëthius) c 475 CE, who is credited with the first literature on teaching the four mathematical sciences: arithmetic, music, geometry, and astronomy. He was deeply intrigued by the philosophical aspect of the study of arithmetic and music. Boethius contributed a body of writing on mathematics that greatly influenced the intellectual evolution of the West during the high Middle Ages, about 1000 CE – 1250 CE. He was not that interested in creating new concepts. On the contrary, he was fascinated by the idea of finding eternal laws in the universe.

De Institutione Musica is an essay written by Boethius and sometimes referred to as his musical treatise. It was very popular among philosophers and music theorists of the Baroque and Renaissance periods due to its focus on the interconnectedness of music, planetary motions, ethics, mind, soul and an inclusive philosophy of harmony. He classified music in three parts:

Musica mundana – music of the world; to be understood without being heard

Musica humana – body/mind/spirit harmony

Musica instrumentalis – instrumental music

The intertwining of arithmetic and music, to him, provided the basis for understanding the universe. Unfortunately he was imprisoned and executed for treason as a result of his efforts to confront the corruption found in the Roman courts, and according to the Stanford Encyclopedia of Philosophy, he was also accused of engaging in magic. However, during his final years Boethius wrote the *Consolation of Philosophy,* for which he is most well known.

In the final days of my sabbatical, I happened upon *The Cambridge History of Western Music Theory,* Cambridge University Press 2008. This book was somehow made available online as a free 1002 page PDF. Not only is there a chapter that specifically goes into great detail on music theory and mathematics, a quick review of its entire contents reveals a very thorough and deeply researched account of all the topics I have been exploring in this project. In section 5*, The transmission of ancient music theory into the Middle Ages* by contributing author Calvin M. Bower, I found this reference to the work of Marcus Tullius Cicero and the platonic traditions; "The ratios that governed the highest order of the physical universe and the metaphysical world itself were those that determined musical concord, and the degree to which sensual music was shaped by these ratios, was the degree to which the soul was led away from rank sensuality to contemplate eternal truths." I believe that developing a greater understanding of the history of music theory from this perspective will provide a rich and meaningful foundation for the integration of math & music in course content.

All of the sciences as we know them today have been related to number, and in some sense, to music. The age-old connection between music and science dates back to the ancient Greeks. It was Philolaus in the fourth century BC, who stated, “All things which can be known have number; for it is not possible that without number anything can either be conceived or known.”

Throughout time, thinkers and those who were philosophically inclined have recognized some connection between the mechanisms of the physical universe, especially the motions of celestial bodies, and the esthetics of musical harmony and rhythm. Beginning with Pythagoras who thought that the Sun, the Moon and the visible planets "hummed" as they pursued their paths in the solar system and continuing on through Johannes Kepler's Latin work *Harmonices Mundi*, (The Harmony of the World) in 1619, to the masterpiece by Guy Murchie in 1961, *Music of the Spheres*, music and science have been seen as part of one organic whole.

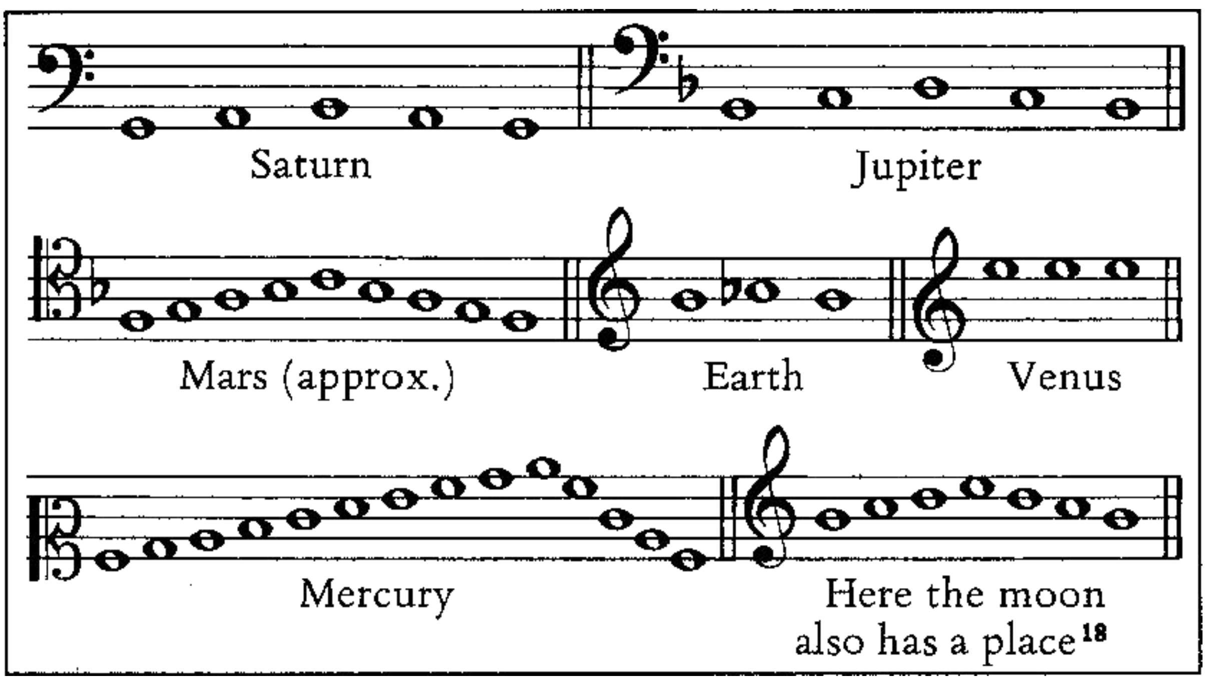
By Kepler's time, astronomy had already been separated from astrology, largely because of the invention of the telescope to which Kepler contributed greatly. With this new tool and his better understanding and his discovery of the so-called "third law of planetary motion," Kepler saw musical harmony in geometrical forms and physical phenomena. He had a keen interest in music, which included familiarity with great composers of his time, and he had an almost mystical belief in what he called "the mournful song" of the Earth's motion around the sun.


HARMONICIS LIB. V. 207

mnia (infinita in potentiâ) permeantes actu : id quod aliter à me non CAP. VI
potuit exprimi, quam per continuam ſeriem Notarum intermedia-

rum. Venus ferè manet in uniſono non æquans tenſionis amplitu-
dine vel minimum ex concinnis intervallis.

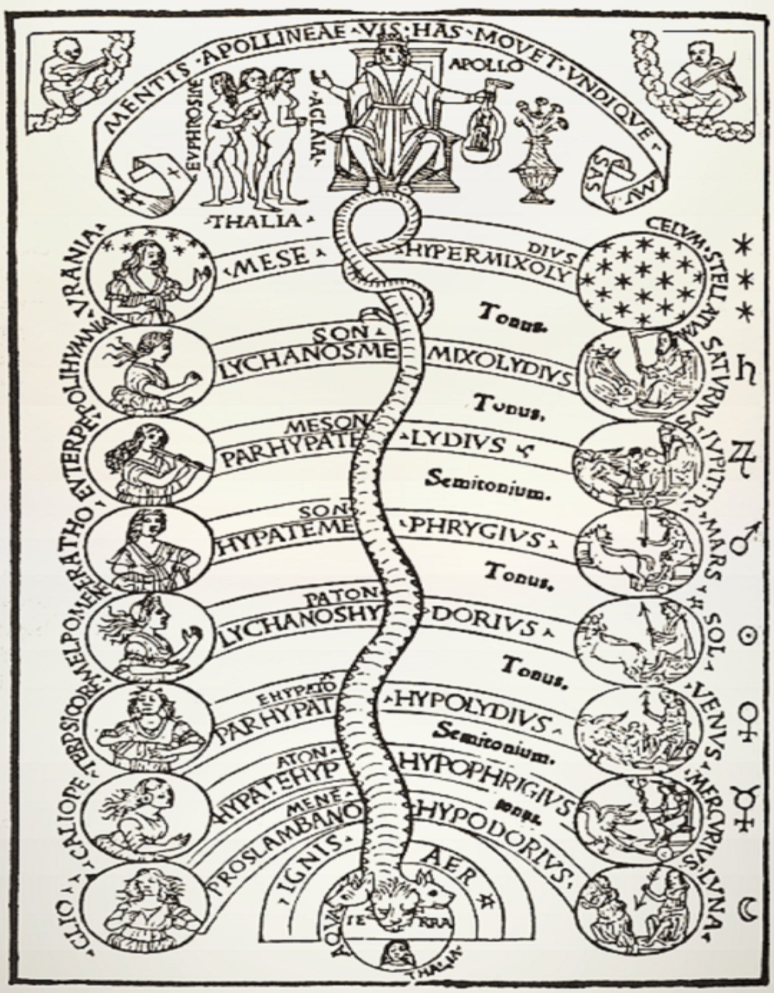
Kepler's notation (copy of a copy from the original text of *Harmonices Mundi)*



A modern reproduction of Kepler's image

Kepler's ideas influenced Isaac Newton, who had studied musical text and once commented, "Pythagoras's Musick of the Spheres was gravity." In 1961, Guy Murchie wrote *Music of the Spheres,* which followed his *Song of the Sky* in 1954. Both these books were combinations of scientific cleverness and musical understanding and appreciation. Both built upon and speculated about the age-old link between the hard data of the physical world and the ethereal beauty of music.

Apollo, the Muses, the planetary spheres and musical modes



The Music of the Spheres. From Francinus Gafurius, *Practica Musice* (Florence, 1496).

https://commons.wikimedia.org/wiki/File:The_music_of_the_spheres.jpg -

https://joedubs.com/the-platonic-and-pythagorean-solids/

Part III: The Power of Music

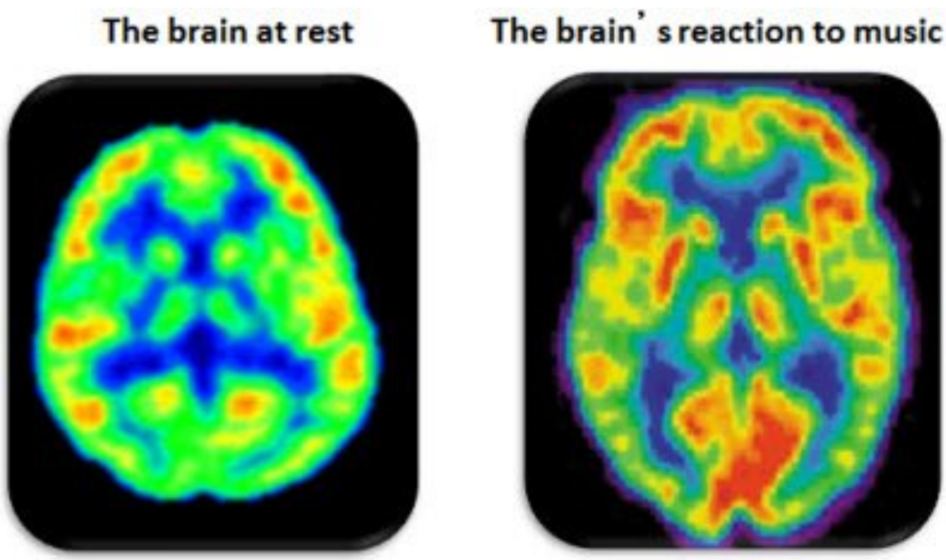
Back to ol' Pythagoras! The power of music was so accepted by the ancients that it has been noted that they used certain melodies to promote restful sleep. Different musical modes were used depending on what was needed and so upon rising after a deep sleep, in order to leave behind the stupor, they would listen to music in a mode that would stimulate alertness. Pythagoras taught that you could heal by using sound and harmonic frequencies and he is believed to be the first person to prescribe music as medicine.

In the book, *This is your Brain on Music*, Daniel J. Levitin takes us on a journey into the modern research and analysis of what may have already been believed in ancient Greece. He states, "From a one-dimensional continuum of molecules vibrating at different speeds, our brains construct a rich, multidimensional pitch space with three, four, or even five dimensions (according to some models). If our brain is adding this many dimensions to what is out there in the world, this can help explain the deep reactions we have to sounds that are properly constructed and skillfully combined."

Levitin provides a summary of music history, of how we are affected from a neuroscience perspective, and how we evolved to have musical preferences. He makes reference to research on fetuses and how, according to Alexandra Lamont, co-author of *The Psychology of Musical Development*, at one year of age a baby will recognize a song they heard in utero. Levitin supports this by saying that our brains are imprinted by every song we hear. Most interesting to me is when he elaborates quite a bit on the subconscious predictions that we are making when listening to music and how this determines how much or how little we enjoy the sound. I can say that I don't entirely agree with that. From my personal experience, in particular when listening to the music of J.S. Bach, I have experienced something quite different. For example, the first time I listened to Bach's *Prelude from Partita BWV 997*, there was nothing at all predictable and in fact it almost sounded like the opening notes were from one of the modes that I am not accustomed to listening to. It sounded peculiar and as if the performer might have been making mistakes. Yet, I was extremely drawn into the music. It was quite pleasing to me and unlike anything I had ever heard and I was compelled to listen all the way through. This will need to be explored further.

Levitin reminds us that while some scientists argue that the sole purpose of music is simply for pleasure seeking, most of the scientific community will agree that music played a significant role in the development of human speech and language.

The author explains “multiple trace theory” and how we can look at brain scans that show areas of the brain lighting up while listening to music. The scans show how we process the abstract information such as melody and timbre, compared to the more specific information like the vocabulary used in the lyrics, when listening to music. Levitin tells us that these brain scan studies can show us how an old childhood memory can be triggered by hearing a song from that time period.



The Mozart effect, or not?

Although I would love to believe that listening to classical music makes a person smarter, the truth is that the research clearly shows that there is not enough evidence, and the claim is considered controversial. If it were true, I suspect that my own IQ would be nearly 250 because of all the Bach, Mozart, Beethoven, Chopin, and Scarlatti I have listened to! The most legitimate source I found on *The Mozart Effect*, is from the NCBI database. The National Center for Biotechnology Information advances science and health by providing access to biomedical and genomic information. *The Mozart effect, J.S. Jenkins JRSM, 2001,* is a publication from the UK based Journal of the Royal Society of Medicine and is the result of scientific study.

The study explains the brain science and gives some simple examples that are easy to understand such as which parts of the brain are responsible for specific aspects of music like pitch, timbre, and rhythm. It provides details of the experiments and does point out that for some patients with epilepsy there was a favorable response to the sound of Mozart's piano sonata K448. Some had a significant improvement. Brain "spikes" associated with seizure activity had decreased significantly.

The study even included experiments with rats that were exposed to Mozart's K448 Sonata, white noise, silence, and the music of contemporary minimalist composer Philip Glass. They used four samples that were aesthetically very different sounding. This is because some critics of the theory claimed that the improvement seen in previous experiments (with human subjects) listening to Mozart, was due to the "enjoyment arousal" factor.

Although some results of the Jenkins study showed an improvement in spatial-temporal reasoning in a few of the subjects, the conclusion is that there is not enough evidence to prove that listening to classical music can improve intelligence or scholastic aptitude. However, it seems to me that creating an environment such as a classroom, with a component that is likely to be enjoyable would certainly increase the chances of student engagement and possibly a more relaxed and open mind. I have in fact experimented with 5 minutes of music appreciation at the beginning of math class, and it had obvious positive effects. It wasn't Mozart or Bach, but in the near future I am planning on a more focused and intentional use of the music appreciation activity.

Part IV: Music in the Math Classroom

"Philosophy is written in this grand book, the universe, which stands continually open to our gaze. But the book cannot be understood unless one first learns to comprehend the language and read the letters in which it is composed. It is written in the language of mathematics, and its characters are triangles, circles, and other geometric figures without which it is humanly impossible to understand a single word of it; without these, one wanders about in a dark labyrinth." Galileo

Rhythm

Introductory music theory

The science of sound

Tuning and temperaments

12 tone music

Symmetry in music

Mathematical modern music

This list of music topics is the beginning stage of what I would like to develop into a supplemental curriculum for my 11-week basic math skills class for adults. In the last section of my sabbatical project I looked at what other instructors and experts have been doing to improve math instruction specifically with musical connections. First, I contacted local elementary schools, and ABSE instructors from nearby community colleges and did not find anyone who is currently doing this or anything even similar. I feel we need to devote some time and energy into reviving the ways of the Quadrivium.

We need to accelerate our creative energies and make an effort to expand the much needed basic math curriculum built upon state approved learning standards by integrating music as well as art, nature, health, and philosophy.

Gareth E. Roberts, Mathematics professor at the College of the Holy Cross in Worcester, MA, published and implemented his mathematics and music course with success and positive outcomes. He explains that in his experience, most students who do poorly in the beginning level math classes in high school are then not able to access other more interesting math topics. Roberts says that in his mathematics and music course, "students who did not think of themselves as mathematically inclined have unearthed hidden talents and interests in the area." The title of his 320-page book *From Music to Mathematics: Exploring the Connections* from the publishers of Johns Hopkins University Press is nearly the same as the title of my project and will surely be on my summer reading list. Although curriculum development is not the purpose of this sabbatical, I have certainly found some interesting places to start and samples of lesson ideas that are quite a bit more interesting than what I had experimented with in the didgeridoo class project.

In *Music: A Mathematical Offering* by Dave Benson, the author goes into depth on the science of sound and the way the human ear works. He teaches us in great detail about sine waves, periodicity, the harmonics of vibrating strings and much more. This is essentially a manual on the physical science of sound and music, complete with the complex mathematical equations that explain the behaviors of various musical instruments in an orchestra. He provides examples of wave motions, and what is meant by acoustic pressure and displacement in wind instruments. Benson's book is fascinating with its vastness of scientific knowledge, many examples, illustrations, and multicultural perspectives. Following each section, there is an exercise for the reader. For example, after describing the vibrational patterns on drum membranes, complete with diagrams, equations, and ratios, he asks, "What does a square drum sound like?"

Although most of the math and science in Benson's book is at a level that is difficult for my understanding, I have been able to extract some ideas that can be useful in my future math and music course content. The following are examples of musical concepts that could be built into a math lesson on symmetry.

Reflectional symmetry appears in music in the form of inversion of a figure or phrase.

The lower line is obtained by inverting the upper line.

This is the musical equivalent of the palindrome.

Each bar of the upper line of the left hand is inverted to form the next bar.

Helpful Research

To learn more about how I can develop my ideas, I am very happy to have a found an extensive peer reviewed research study that was published in 2013. The study, *Elementary Teachers Integrate Music Activities into Regular Mathematics Lessons: Effects on Students' Mathematical Abilities,* investigated the integration of music into mathematics lessons and was examined with pre and post testing. The effects of the integrated curriculum clearly showed positive results in a variety of mathematical abilities.

The authors of this research study began their investigation as a result of the body of research that presents a convincing case against traditional math curriculum and teaching methods. They went to the work of Garner's multiple intelligences and several other motivational theorists for the framework and then designed and administered an intervention involving two elementary school teachers with similar demographic backgrounds, from two separate schools. The students in each of their classes were also demographically similar. Over a five- week period, the students participated in activities that integrated various music concepts such as music composition, with the usual math lessons. The content of these lessons included number sense, mathematical reasoning, measurement, statistics, and other topics that are also taught in ABSE Math 1. The research design tools, procedures, and data collection and analysis seem so well done that it inspires and encourages me to move forward in my plan of bringing music to the math class I am currently teaching. However, it is important to note that I will need to work on adaptations and modifications that take into consideration the differences in the learning needs between children and adults. I will be revisiting this study when I begin my Math & Music curriculum work, as the research report includes the lessons and activities carried out by the teachers.

Adult Learning

Common sense and conventional wisdom have long held that children are better learners than adults. After all, for more than three centuries Jean-Jacques Rousseau's “tabula rasa” concept, the "empty slate," dominated educational theory and even today still seems hard to refute. Children's "slates" are relatively empty while adults have plenty written on theirs, and moreover, a lot of it is just plain wrong. If learning is conceived of as writing on an empty chalkboard, then there is little room for adults to learn because their "chalkboards" are relatively full while the child's empty slate is practically begging to be written upon. But recent research has challenged the tabula rasa view, and in fact, adults may learn as well as, if not better than children.

Adult Learning Theory: Evolution and Future Directions by Shanan B. Merriam, is an in depth analysis of adult learning and includes a thorough review of the three major theories that have evolved, and the more recent contexts that are the foundations of understanding the learning needs of adults in today's adult education classrooms. Merriam's work is most relevant to me in my exploration because she writes about the holistic elements of teaching and learning with the emphasis on creativity, the arts, health and well being. She is an advocate of transformative learning and reports that this area of adult learning theory is backed with an ample supply of researched based literature including a journal of international conferences. In her claim that a holistic approach should include the spiritual dimensions of the human being, she makes a clear distinction between religion and spirituality and offers some opinion as to why the role of spirituality in learning has not been widely accepted. Merriam also comments on the multi-cultural perspectives of learning that are in contrast with the formal schooling of the West and its rational cognitive framework. She states, "Our body, our emotions, and our spirit (what is often referred to as holistic learning), are also important avenues for learning or knowledge construction." In concluding, the author provides a compelling argument for this holistic view as she also writes about meaning-making and how adults enter a classroom with a "meaning-making agenda". In defining this, she specifically points to the role of images and symbols in the construction of knowledge and regarding the images and symbols she states, "which often emanate from the deepest core of our being and can be accessed and manifested through art, music, or other creative work."

In closing I will add that based on my renewed understanding, and on some of the literature that I am planning to study, my hopes for combining music and math will involve more than just the learning process as students work toward their goals in college. Integrating the arts in general, and teaching from a perspective of the

Quadrivium and the elements of classical education will, in my opinion, provide a stronger foundation for instilling the value of life-long learning in our students. More specifically, the combination of music and math in ABSE basic skills is unique, and as a result of this sabbatical work, my plans for moving forward in developing my ideas and future course content are well supported.

School Observations

After several attempts to visit local schools where teachers may be integrating music and math instruction, I was unable to find anyone who is presently doing something that would be related to this project. I contacted 3 elementary schools in the 4J district as well as ten ABSE instructors from community colleges around the states of Oregon and Washington. Most of the ABSE instructors, in one way or another, replied by letting me know that although they were not currently integrating music and math they found my project to be very interesting. The administrator of one of the Eugene elementary schools responded positively, informing me that the inquiry would be passed on to instructors. After a second attempt to connect with that school, I still received no replies. I am assuming that they are simply too busy!

Musicianship

My competence as a musician and my understanding of its theory and aesthetics is crucial to my continuing plan of integrating music and math. To continue my growth and foundation for the project, I have committed to daily music practice and for this sabbatical this was proposed as 2-3 hours each day.

Notes on my weekly music practice

Week 1: Set goals

1. Build my classical repertoire
   Learn one new piece every 2-3 weeks
   Improve technique as weaknesses are discovered
2. Improve pitch and tone on the cello
3. Study: Baroque fugue
   African rhythms
   Classical Indian music

Week 2:

Learned new guitar piece: *Rondeau in A* by David Kellner, 1670 – 1748

- Enjoyable to play
- Needs to be performed at a faster tempo than I am accustomed to

Worked on pitch with the cello

- Need to eliminate the sheet music and just focus on sound

Week 3:

Improved cello bow speed
Worked on tempo with guitar
In progress: Understanding the fugue
Listened to and analyzed J.S. Bach - *Fugue BWV 1001* arranged for classical guitar

Week 4:

Learned new guitar piece: *Adelita* by Francisco Tàrrega, 1852 - 1909

- Difficult due to mordents that require great accuracy
- Sounds best at a slower tempo

Understanding the fugue

- Watched an instructional video on how to compose a fugue
- Questioning math concepts that can be applied

Week 5:

Worked on pitch with the cello

- Play scales along with pre-recorded cello drones

Study the technique of producing right hand harmonics on the guitar

- Watched several instructional videos

Listened to and analyzed J.S. Bach - *Toccata and Fugue in D minor BWV 565*

Week 6:

Improved cello bow speed and control

- Learned to "dive in" deeper into the strings

Listened to North African rhythms

- DRUM & African Rhythm, Part 1 | Mokhtar Samba

Watched video- Music is Math! Teaching Rhythms as Fractions to Middle Elementary

Week 7:

Began a volunteer position at RiverBend Hospital in the *Sounds for Healing* program

- Weekly performance on guitar in the hospital lobby-- "*To create a healing and relaxing environment for patients and families … through the gift of music.*"

In progress: Reading *The Classical Indian Just Intonation Tuning System*

Week 8:

Extensive rehearsal of my guitar repertoire in preparation for RiverBend Hospital

- Select all pieces that are suitable for the *Sounds for Healing* program
- Add my own arrangements of some pop music

Continued perfecting the right hand harmonics

Listened to Anoushka Shankar - Indian Classical Raga

Week 9:

Learned new guitar piece: *Gymnopèdie No. 1* by Erik Satie, 1866 – 1925

- Sounds best when slightly flattening the b and g strings
- Produces a meditative state
- Added right hand harmonics to the ending

Week 10:

Watched a video, *J.S. Bach - The Art Of Fugue BWV 1080*

First attempt at composing a simple fugue

Continued study of classical Indian music

- Explored melodic patterns to use in original composition

Watched video- Mathematics of African Dance Rhythms

Week 11:

Began learning new guitar piece: *Moon River* by Henry Mancini, 1924 – 1994

- Easy and enjoyable
- Especially appropriate for the *Sounds for Healing* program

Composing a fugue -- *to be continued…*

References

An, Capraro, & Tillman 2013, *Elementary Teachers Integrate Music Activities into Regular Mathematics Lessons: Effects on Students' Mathematical Abilities*
Journal for Learning through the Arts
https://files.eric.ed.gov/fulltext/EJ1018326.pdf

Benson, Dave 2008, *Music: A Mathematical Offering*
https://homepages.abdn.ac.uk/d.j.benson/pages/html/maths-music.html

Blaschke, Lisa Marie 2012, *Heutagogy and Lifelong Learning: A Review of Heutagogical Practice and Self-Determined Learning,*
http://www.irrodl.org/index.php/irrodl/article/view/1076/2087

CALM: *Curriculum For Adults Learning Math*
https://external-wiki.terc.edu/display/CALM

Christensen, Thomas 2008, *The Cambridge History of Western Music Theory*
https://www.cambridge.org/core/books/cambridge-history-of-western-music-theory/1E331E53DC92BE497C74D40DE195105A

Drake, Stillman 1957, *THE ASSAYER, GALILEO GALILEI 1623*
Selections translated by Stillman Drake, *Discoveries and Opinions of Galileo*
https://www.princeton.edu/~hos/h291/assayer.htm

Encyclopaedia Britannica, *Kepler's laws of planetary motion*
https://www.britannica.com/science/Keplers-laws-of-planetary-motion

Galileo quote
http://math.furman.edu/~mwoodard/ascquotg.html

Godwin, Joscelyn 2017, *Kepler and Kircher on the Harmony of the Spheres*
https://hermetic.com/godwin/kepler-and-kircher-on-the-harmony-of-the-spheres

Hornbeck, David 2013, *Mathematics, Music, and the Guitar,* David Hornbeck, 2013
http://jwilson.coe.uga.edu/EMAT6500/EMAT6500.html

Jenkins, J.S. 2001, *Mozart Effect JRSM*
https://www.ncbi.nlm.nih.gov/pmc/articles/PMC1281386/

Levitin, Daniel J. 2006, *This is Your Brain on Music: The Science of a Human Obsession*

Merriam, Sharan B. 2017, *Adult Learning Theory: Evolution and Future Directions*
PAACE Journal of Lifelong Learning, Vol. 26, 2017, 21-37
https://www.iup.edu/ace/paace/
https://www.iup.edu/WorkArea/DownloadAsset.aspx?id=250023

Mosby, Todd 2014, *Boethius And The Relevance Of Musical Understanding*
https://toddmosbymusic.com/essays/boethius-and-the-relevance-of-musical-understanding

Nonesuch, Kate 2006, *Changing the Way we Teach Math*
Nonesuch, Kate 2006, *More Complicated than it Seems*
https://katenonesuch.com/resources/math/

Perrin, Christopher A. 2004, *An Introduction to Classical Education: A Guide for Parents*
http://www.ClassicalAcademicPress.com

Prins & Vanhaelen 2017, *Sing Aloud Harmonious Spheres: Renaissance Conceptions of the Pythagorean Music of the Universe*

Rees, Torben 2009, *Monochord: An Ancient Musical and Scientific Instrument*
http://www.sites.hps.cam.ac.uk/whipple/explore/acoustics/monochord/

Roberts, Gareth E., Johns Hopkins University Press 2016, *From Music to Mathematics: Exploring the Connections*
http://mathcs.holycross.edu/~groberts/Talks/HCAlum08Web.pdf

Rosenthal, Jeffrey 2005, *The Magical Mathematics of Music,*
https://plus.maths.org/content/magical-mathematics-music

Society for Industrial and Applied Mathematics
https://archive.siam.org/careers/pdf/guitar.pdf

Sorensen, Lars 2008, *Mozart on the Brain: Musical Misadventures in Cognition and Development, Cognition & Language: Birth to Eight*,
https://www.cs.rutgers.edu/~biglars/Mozart.html

Stanford Encyclopedia of Philosophy 2016, *Anicius Manlius Severinus Boethius*
https://plato.stanford.edu/entries/boethius/#PrimTextLati

Stanford Encyclopedia of Philosophy, 2017, *Jean Jacques Rousseau*
https://plato.stanford.edu/entries/rousseau/

STEM Guitar Building Project
http://www.guitarbuilding.org/

Weiss, Susan Forscher 2010, *Musical Education in the Middle Ages & Renaissance*
https://newbooksinpolitics.com/political/music-education-in-the-middle-ages-and-the-renaissance/